Below is a list of statements some of which are false, and some of which are true. Decide which is which.

In what follows $A$ is an arbitrary $n \times n$ matrix.

1. Every $2 \times 2$ matrix is diagonalizable.
2. If $A^{2011}$ is invertible, then so is $A$ (Hint: compute det).
3. The determinant of the block matrix $\begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ equals $\det(A) \det(D)$.
4. $\det(AB) = \det(A) \det(B)$.
5. If $A$, $B$ are matrices with complex entries, then $\overline{AB} = \overline{A} \overline{B}$
6. If $A$, $B$, $C$ are $n \times n$ matrices, then $AB = AC$ implies $B = C$.
7. If $A$, $B$, $C$ are $n \times n$ matrices, then $A[B, C] = [AB, AC]$.
8. If the product of two matrices is $I_n$, then both matrices are invertible. (Decide whether this is true for arbitrary matrices, and then for square matrices).
9. If $A$ and $AB$ are invertible, then $B$ is invertible.
10. If $A^\top$ is invertible, then $A$ is invertible.
11. the dimension of $\text{Nul}A$ is the number of free variables in solving $AX = 0$.
12. any two bases of a subspace $H$ have the same number of vectors.
13. the subspaces $\text{Col}A$ and $\text{Col}R$ have the same dimension.
14. the subspaces $\text{Nul}A$ and $\text{Nul}R$ are equal.
15. the subspaces $\text{Col}A$ and $\text{Col}R$ are equal.
16. Pivotal columns form a basis of $\text{Col}A$.
17. $\det(A) = 0$ if and only if $A$ is invertible.
18. $\lambda$ is an eigenvalue of the block matrix $\begin{bmatrix} A & B \\ 0 & D \end{bmatrix}$ if and only if $\lambda$ is an eigenvalue of $A$ or $D$.

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20. If $A$ is a (upper or lower) triangular matrix with entry $a_{ij}$ at the $ij$ slot, then the eigenvalues of $A$ are $a_{11}, \ldots, a_{nn}$.

21. If $A = LU$ is the LU factorization of $A$, then $U$ and $A$ have the same determinant.

22. If $A = LU$ is the LU factorization of $A$, then $U$ and $A$ have the same eigenvalues.

23. $A$ is diagonalizable if and only if eigenvalues of $A$ form a basis.

24. If all eigenvalues of $A$ are real, then $A$ is symmetric.

25. $\det(A) = \lambda_1 \ldots \lambda_n$.

26. $\det(A - \lambda I_n) = (\lambda_1 - \lambda) \ldots (\lambda_n - \lambda)$ where $\lambda_1, \ldots, \lambda_n$ are eigenvalues of $A$.

27. If $A$ is symmetric (that is $A^\top = A$), then the eigenvalues of $A$ are real.

28. If $A$ is diagonalizable, then $A$ has $n$ distinct eigenvalues.

29. $A$ has $n$ eigenvalues (counted with multiplicities, that is possibly some of them are equal).

30. $\lambda$ is an eigenvalue of $A$ if and only if $A - \lambda I_n$ is not invertible.

31. If $v$ is an eigenvector for the eigenvalue $\lambda$, then $\bar{v}$ is an eigenvector for $\bar{\lambda}$.

32. If $\lambda$ is an eigenvalue of $A$, then so is $\bar{\lambda}$, the complex conjugate of $\lambda$.

33. The matrices $B^{-1}AB$ and $A$ have the same eigenvectors.

34. The matrices $B^{-1}AB$ and $A$ have the same eigenvalues.

35. If $P$ is an $n \times n$ matrix whose columns are eigenvectors of $A$, then $AP = PD$ where $D$ is diagonal.
36. If $P$ is an $n \times n$ matrix whose columns are eigenvectors of $A$, then $P$ is invertible.

37. If $A$ is diagonalizable and $B$ is invertible, then $B^{-1}AB$ is diagonalizable.

38. If $A$ is diagonalizable, then so is $A^k$ for all positive integers $k$.

39. If $A^{-1}$ is diagonalizable, then so is $A$.

40. If $v$ is an eigenvector for $A$, and $c$ is a nonzero number, then $cv$ is an eigenvector for $A$.

41. If $v_1$, $v_2$ are eigenvectors for $A$, then $v_1 + v_2$ is an eigenvector for $A$. 